EQUATION REDUCIBLE TO HOMOGENOUS

Ex $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$

Put $x = x'+h$ and $y = y'+k$

$\Rightarrow \frac{dy}{dx} = \frac{dy'}{dx'}$

$\Rightarrow \frac{dy'}{dx'} = \frac{x'+h+2y'+2k-3}{2x'+2h+y'+k-3}$

$= \frac{x'+2y'+(h+2k-3)}{2x'+y'+(2h+k-3)}$

I have to find h and k so that

$h+2k = 3 \quad \times 2 \quad = \quad 2h+4k = 6$

$2h+k = 3 \qquad\qquad\quad 2h+k = 3$

$\Rightarrow \quad 3k = 3 \Rightarrow k = 1$

$\Rightarrow 2h = 2 \Rightarrow h = 1$

$\Rightarrow \frac{dy'}{dx'} = \frac{x'+2y'}{2x'+y'}$

Put $y' = vx'$

$\Rightarrow \frac{dy'}{dx'} = v + x'\frac{dv}{dx'}$

$\Rightarrow v + x'\frac{dv}{dx'} = \frac{x'+2vx'}{2x'+vx'} = \frac{1+2v}{2+v}$

$\Rightarrow x'\frac{dv}{dx'} = \frac{1+2v-v}{2+v} = \frac{1+2v-2v-v^2}{v+2} = \frac{1-v^2}{2+v}$

$\Rightarrow \frac{2+v}{1-v^2}\cdot dv = \frac{dx'}{x'}$

$$\Rightarrow 2\int \frac{dv}{1-v^2} + \frac{(-1)}{2}\int \frac{-2v}{1-v^2}\,dv = \int \frac{dx'}{x'}$$

$$\Rightarrow 2\cdot\frac{1}{2}\log\left|\frac{1+v}{1-v}\right| - \frac{1}{2}\log(1-v^2) = \log x' + c$$

$$\Rightarrow \log\left|\frac{1+\frac{y'}{x'}}{1-\frac{y'}{x'}}\right| - \frac{1}{2}\log\left|1-\frac{y'^2}{x'^2}\right| = \log x' + c$$

$$\Rightarrow \log\left|\frac{x'+y'}{x'-y'}\right| - \frac{1}{2}\log\left|\frac{x'^2-y'^2}{x'^2}\right| = \log x' + c$$

$$\Rightarrow \log\left|\frac{(x-1)+(y-1)}{(x-1)-(y-1)}\right| - \frac{1}{2}\log\left|\frac{(x-1)^2-(y-1)^2}{(x-1)^2}\right| = \log(x-1)^2 + c$$

$$\Rightarrow \log\left|\frac{x+y-2}{x-y-}\right| - \frac{1}{2}\log\{(x-1)^2-(y-1)^2\} + \log(x-1) = \log(x-1)^2 + c$$

$$\Rightarrow \log(x+y-2) - \log(x-y) - \frac{1}{2}\log\{(x-1)^2-(y-1)^2\} = c$$

$$\Rightarrow \log(x+y-2) - \log(x-y) - \frac{1}{2}\log(x+y-2) - \frac{1}{2}\log(x-y)$$

$$\Rightarrow \frac{1}{2}\log(x+y-2) - \frac{3}{2}\log(x-y) = c$$

$$\Rightarrow \log(x+y-2) - 3\log(x-y)^3 = 2c$$

$$\Rightarrow \log(x+y-2) - \log(x-y)^3 = -\log k$$

$$\Rightarrow \log(x+y-2) + \log k = \log(x-y)^3$$

$$\Rightarrow \log k(x+y-2) = \log(x-y)^3$$

$$\Rightarrow k(x+y-2) = (x-y)^3 \quad \text{Ans//}$$

2. $\frac{dy}{dx} = \frac{6x-2y-7}{3x-y+4}$

$\Rightarrow \frac{dy}{dx} = \frac{2(3x-y)-7}{3x-y+4}$

Put $3x-y = z$

$\Rightarrow 3-\frac{dy}{dx} = \frac{dz}{dx}$

$\Rightarrow 3-\frac{dz}{dx} = \frac{dy}{dx}$

$\Rightarrow 3-\frac{dz}{dx} = \frac{2z-7}{z+4}$

$\Rightarrow 3-\frac{2z-7}{z+4} = \frac{dz}{dx}$

$\Rightarrow \frac{3z+12-2z+7}{z+4} = \frac{dz}{dx}$

$\Rightarrow \frac{z+19}{z+4} = \frac{dz}{dx}$

$\Rightarrow \frac{z+4}{z+19} \cdot dz = dx$

$\Rightarrow \frac{(z+19-15)}{z+19} dz = dx$

$\Rightarrow \int dz - 15\int \frac{dz}{z+19} = \int dx$

$\Rightarrow z - 15\log(z+19) = x + c$

$\Rightarrow 3x-y-15\log(3x-y+19) = x+c$

$\Rightarrow 2x-y-c = 15\log(3x-y+19)$ Ans